Saint Mary's University

Department of Mathematics
& Computing Science

MATH 3441 – Real Analysis I

Fall 2022

Territorial Acknowledgement

Saint Mary's University acknowledges that the university is located in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People.
This territory is covered by the "Treaties of Peace and Friendship" which Mi'kmaq, Wəlastəkwiyik (Maliseet), and Passamaquoddy Peoples first signed with the British Crown in 1726.
The treaties did not deal with surrender of lands and resources but in fact recognized Mi'kmaq and Wəlastəkwiyik (Maliseet) title and established the rules for what was to be an ongoing relationship between nations.

Instructor: Dr. Manuela Girotti

Office: McNally North MN 106

Email: `manuela.girotti@smu.ca`

Please put "MATH 3441" in the subject line, use the *plain text format*, and make sure that you are clearly identified (first and last names). I do not answer anonymous email. I do not check emails during evenings or weekends. I usually answer during the first business day after receiving an email.

Lectures: Synchronous.

Mondays and Wednesdays, 1:00pm–2:15pm (Halifax time)

Loyola Academic (LA), room 280

Office hours: TBD.

Overview: The emphasis will be on **metric spaces**. We will discuss in detail concepts such as: open and closed sets, boundedness, total boundedness, compactness, sequences, completeness, continuity, uniform continuity, sequences of functions, pointwise and uniform convergence, metric spaces of functions. Time permitting, we will cover additionally the theorems of Baire, Ascoli-Arzelà, and Stone–Weierstrass.

Prerequisites: good knowledge of Introductory Analysis (MATH 2310).

Textbook: Pre-started Class Notes (posted on Brightspace) and class notes. The notes are for your use only - do not share them with anyone!

Other useful resources:

- M. Ó Searcóid, *Metric spaces*, Springer Undergraduate Mathematics Series, Springer, 2007.
- S. Shirali and H. L. Vasudeva, *Metric Spaces*, Springer, 2006.
- W. Rudin, *Principles of Mathematical Analysis*, International Series in Pure&Applied Mathematics, McGraw-Hill, 1976.

A diary of the lectures will be regularly kept on the Brightspace calendar with the sections covered in each class. Please, refer to that when preparing for the final exam, as that will be the official and ultimate syllabus for the class.

Evaluations: The course mark will be calculated as follows:

30%	homework assignments,
5%	*active* in-class participation,
20%	quizzes,
45%	final exam.

Note that there is no "100% final exam" option in this course. The term work contributes 55% to the final grade. Therefore, active participation in classes and continuous work on the course material during the semester is essential for success in this course!

The final score will be out of 100 and the breakdown of the grades is the following:

Grade	F	D	C	C+	B-	B	B+	A-	A	A+
Percentage	0-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-100

Homework: You will be required to hand in about **9 assignments** along the semester (approximately one every week).

Assignments are very important! Taking them seriously and doing them well is by far the best way to learn. Make sure that you start working on them well before the deadline. One day will usually not be enough. You should at least read the questions on the day the assignment is posted. Before you attempt a question write out relevant definitions and results. Do not hand in your rough work. Always use full sentences. Solutions that are not legible will not be marked.

The assignments will be posted on Brightspace and on Crowdmark and they reflect the content of the course.

No late assignments will be accepted.

Don't forget to write your name and SMU ID on the first page and to sign it.

To submit your assignment you can either

- write it on paper (filling the empty spaces provided on the assignment) and scan it;
- type it on the computer by using LaTeX, Overleaf or other softwares that support Mathematics symbols;
- write it on your tablet using a handwriting app.

In all of these cases, you will then need to *upload your homework on Crowdmark.*

Discussions and work group are highly encouraged!

However,

1. **Acknowledge** all people you worked with or got help from (e.g., I worked with John Smith on Problems 2, 3, 5, and got additional help from Jane Doe on Problems 1, 3 and 4.).
   You must also properly acknowledge all other sources you got help from (e.g., textbooks and online sources, like Mathoverflow). If you find a solution somewhere online make sure that the acknowledgement is very precise; in particular this means that you must provide the exact web address for each problem (or part of a problem).
2. **Write** out the solutions in your own words and on your own (if you work in groups you are not allowed to produce template from which you all copy). Do not just copy from a book or an online source.
3. Do not submit anything that you do not **understand**! I reserve the right to quiz you on any part of the assignment you submit.

Quizzes: There will be four (4) short quizzes throughout the semester. Quizzes will typically be about 30 minutes and they will be closed-book. The instructor will give notice about the date of the quiz and what it will cover. These will be communicated at least one week in advance.
The quizzes are intended to encourage the class to regularly review the material, to provide practice for the exams, and to give feedback (both to you the student, and to me the teacher) about your progress.

A good practice for quizzes is to write out definitions and their negations. Try to come up with examples that satisfy definitions and with examples where only a part of the definition under investigation fails. Write out the statements of all major results: think about what happens if one of the assumptions in a theorem is missing. Merely reading the notes is not very useful and should not be referred to as studying.

Final exam: The final exam will be scheduled by the Registrar for sometime during the exam period in December.

The Final may ask for

1. statements of definitions, examples, and results covered in class
2. proofs covered in class
3. problems similar to assignment problems
4. some short problems that you may not have seen before

Some suggestions on how to learn proofs:

- recopy the proofs from class notes
- try to identify where each of the assumptions was used
- try to do the proof from memory; before you compare it to the notes wait a while and then read it carefully (and consider if it makes sense)
- when you are writing proofs use full sentences
- when you are attempting an improvised proof write out all the relevant definitions and results you think may be related to the problem

Make-ups: Please, note the dates for the quizzes and the final on the Brightspace calendar! Alternate arrangements will be discussed only in case a valid medical excuse is provided in a timely fashion and no later then 24 hours after the exam. No special arrangements to accommodate travel that coincides with quizzes or the final will be made.

Expectations: All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Academic Integrity: This course will adhere to the SMU Academic Integrity Policy as found on the Academic Integrity and Student Responsibility page.

Students are expected to do their own work during tests and exams. The following activities, although not exhaustive, are examples of activities that are prohibited:

- Copying from another student;
- Allowing another student to copy from you;
- Using unauthorized aids, including: sheets, cell phones and calculators, during test or exam;
- Getting aid from or giving aid to another student during tests and exams;
- Having another student write for you or writing for another student.

Offenders are subject to discipline. Students are urged to read the Academic Integrity Handbook.

An incident of academic dishonesty can have extremely negative consequences: it could delay or bar a student from graduating. A note on a transcript referring to academic dishonesty could very well bar a student from graduate school or affect job opportunities.

This course is a precious opportunity for you to learn something new and valuable. It's an investment on your future. Failing to acquire it will sadly be your loss.

Intellectual property: Content belonging to instructors shared in online courses, including, but not limited to, online lectures, course notes, quizzes, assignments, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member.
Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the Academic Regulations.

Disabilities: Saint Mary's University is committed to providing reasonable accommodations for all persons with disabilities. Students with disabilities who need accommodations shall first contact the Fred Smithers Centre before requesting accommodations for this class.

Students who need accommodations in this course must contact the instructor in a timely manner (at least one week before examinations) to discuss needed accommodations.

(Tentative) course calendar:

Week	Topic	Important dates
1 (Sep 6th)	Welcome! Lecture 0. Countable and uncountable sets	Sep 6th – *Labour Day / Fête du travail*
2 (Sep 12th)	Lecture 1. Metric spaces. Open and closed balls in metric spaces. Lecture 2. Normed vector spaces.	Sep 13th – course registration deadline; Sep 16th – course drop deadline
3 (Sep 19th)	Lecture 3. Convergent sequences in a metric space. Cluster points. Lecture 4. Equivalent distances, the bi-Lipschitz condition.	
4 (Sep 26th)	Lecture 5. Separability in a metric space. Lecture 6-7. Morphisms between metric spaces and normed vector spaces.	Sep 30th – *National Day for Truth and Reconciliation*
5 (Oct 3th)	Lecture 8. Interior, closure and boundary of a set in a metric space. Lecture 9-10. Open and closed sets in a metric space.	
6 (Oct 10th)	Lecture 12. Open/closed sets and continuity.	Oct 10th – *Thanksgiving / Action de grâce*
7 (Oct 17th)	Lecture 13. Completeness via Cauchy sequences. Lecture 14. Completeness via the "nested closed balls" property.	
8 (Oct 24th)	Lecture 23 (+notes). Pointwise and uniform convergence for a sequence of functions. The Banach space $C_b(X, \mathbb{R})$. Lecture 15. Sets of type G_δ and F_σ. Baire's Theorem.	
9 (Oct 31st)	Lecture 16. Other versions of Baire's Theorem. Applications. Lecture 17. Totally bounded sets in a metric space.	
10 (Nov 7th)		*** *Fall break* ***
11 (Nov 14th)	Lecture 18. Compactness via open covers. Lecture 19. Sequential compactness and Lebesgue's covering lemma.	Nov 21st – course withdrawal deadline

Week	Topic	Important dates
12 (Nov 21st)	Lecture 20. Equivalent descriptions of compactness. Lecture 21. Continuous functions on compact metric spaces. Uniform continuity.	
13 (Nov 28th)	Lecture 22. The Extreme Value Theorem. Lecture 24. Equicontinuity and total boundedness in $C(X, \mathbb{R})$ (part I).	
14 (Dec 5th)	Lecture 25. Equicontinuity and total boundedness in $C(X, \mathbb{R})$ (part II). Lecture 26 (+notes). Stone–Weiestrass Theorem.	

Disclaimer: the instructor reserves the right to make changes to the course outline and course content should this be necessary for academic or other reasons. Changes will also be posted on Brightspace and promptly communicated. Every effort will be made to minimize such changes.