Saint Mary's University

Department of Mathematics
& Computing Science

MATH 3406 – Differential Equations II

Winter 2022

Territorial Acknowledgement

Saint Mary's University acknowledges that the university is located in Mi'kma'ki, the ancestral and unceded territory of the Mi'kmaq People.
This territory is covered by the "Treaties of Peace and Friendship" which Mi'kmaq, Wəlastəkwiyik (Maliseet), and Passamaquoddy Peoples first signed with the British Crown in 1726.
The treaties did not deal with surrender of lands and resources but in fact recognized Mi'kmaq and Wəlastəkwiyik (Maliseet) title and established the rules for what was to be an ongoing relationship between nations.

Instructor: Dr. Manuela Girotti

Office: McNally North MN 106

Email: `manuela.girotti@smu.ca`

Please put "MATH 3406" in the subject line, use the *plain text format*, and make sure that you are clearly identified (first and last names). I do not answer anonymous email. I do not check emails during evenings or weekends. I usually answer during the first business day after receiving an email.

Lectures: Synchronous.

Mondays and Wednesdays, 1:00pm–2:15pm (Halifax time)

Loyola Academic room 280

Office hours: Mondays and Wednesdays, 11:30am–1:00pm, 2:30pm–4pm.
Fridays, 9:00am–10:00am.

Overview: This course will continue the exploration of (Ordinary) Differential Equations already begun with the course MATH 2303 / MATH 3405.
The main topics will be the following:

- Series solutions for second order ODEs.
- Theory of systems of linear differential equations: linear systems with constant coefficients, solution by matrix methods.
- Nonlinear differential equations: existence and uniqueness of solutions, stability and the phase plane, Lyapunov Method.
- Bifurcation (and Chaos).
- Introduction to Partial Differential Equations.

Every topic will be complemented by the analysis of some real-world applications.

Prerequisites: MATH 2303 or MATH 3405 (Differential Equations I).
Good knowledge of Calculus I and II (MATH 1210 and MATH 1211), Multivariable Calculus (MATH 2311) and Linear Algebra (MATH 2301 or MATH 2320).

Textbook: We will cover part of Chapter 2 and the entirety of Chapters 3, 4 and 5 of Braun's book. Complementary course notes will also be distributed in due time.

References:

- Martin Braun, *Differential Equations and Their Applications. An Introduction to Applied Mathematics*, 4th edition, Springer New York, Texts in Applied Mathematics Series #11 (ISBN 978-1-4612-4360-1).

  Some hardcopies are available at the Patrick Power Library.

- R. Kent Nagle, Edward B. Saff, Arthur David Snider, *Fundamentals of Differential Equations*, 9th edition, Pearson (ISBN 9780321977069).

A diary of the lectures will be regularly kept on the Brightspace calendar with the sections covered in each class. Please, refer to that when preparing for the final exam because that will be the official and ultimate syllabus for the class.

Evaluations: The course mark will be calculated as follows:

30% assignments,
30% two midterm exams,
5% *active* in-class participation,
35% final exam.

Note that there is no "100% final exam" option in this course. The term work contributes 65% to the final grade. Therefore, active participation in classes and continuous work on the course material during the semester is essential for success in this course.
The final score will be out of 100 and the breakdown of the grades is the following:

Grade	F	D	C	C+	B-	B	B+	A-	A	A+
Percentage	0-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-100

Homework: You will be required to hand in about **8 assignments** along the semester (approximately, one every week).
The assignments will be posted on the Brightspace website with due dates and they reflect the content of the course.
No late assignments will be accepted.

Discussions and work group are highly encouraged!

To submit your assignment you can either

- write it on paper (filling the empty spaces provided on the assignment) and scan it;
- type it on the computer by using LaTeX, Overleaf or other softwares that support Mathematics symbols;
- write it on your tablet using a handwriting app.

You will then need to *upload your homework on Crowdmark*.

Midterm exams: There will be **two midterm exams**. The midterms will be **take-home exams**, to be taken during a continuous 2-hour period. The dates and content of the midterms will be communicated at least 10 days in advance.
The examination will be open book: you can use all class material (class notes, homework problems, solutions). It is forbidden to use any other material, to look up solutions online, and to discuss with other peers.

Final exam: The final examination will be a **take-home exam**, to be taken during a continuous 3 hour period.
The final exam due date will be scheduled by the Registrar for some time during the exam period in April.
The final exam will cover material from the entire course and it will be open book: you can use all class material (class notes, homework problems, solutions). It is forbidden to use any other material, to look up solutions online, and to discuss with other peers.

Make-ups: Alternate arrangements will be discussed only in case a valid medical excuse is provided in a timely fashion and no later then 24 hours after the exam. No special arrangements to accommodate travel that coincides with midterms or the final will be made.

Expectations: All individuals participating in courses are expected to be professional and constructive throughout the course, including in their communications.

Calculators: unless otherwise stated, basic 4-function calculators and scientific calculators (like Sharp EL 531 and Casio FX 300MS, for example) are permitted in class tests and final examination.

Academic Integrity:

This course will adhere to the SMU Academic Integrity Policy as found on the Academic Integrity and Student Responsibility page.

Students are expected to do their own work during tests and exams. The following activities, although not exhaustive, are examples of activities that are prohibited:

- Copying from another student;
- Allowing another student to copy from you;
- Using unauthorized aids, including: sheets, cell phones and calculators, during test or exam;
- Getting aid from or giving aid to another student during tests and exams;
- Having another student write for you or writing for another student.

Offenders are subject to discipline. Students are urged to read the Academic Integrity Handbook.

An incident of academic dishonesty can have extremely negative consequences: it could delay or bar a student from graduating. A note on a transcript referring to academic dishonesty could very well bar a student from graduate school or affect job opportunities.

This course is a precious opportunity for you to learn something new and valuable. It's an investment on your future. Failing to acquire it will sadly be your loss.

Intellectual property:

Content belonging to instructors shared in online courses, including, but not limited to, online lectures, course notes, quizzes, assignments, and video recordings of classes remain the intellectual property of the faculty member. It may not be distributed, published or broadcast, in whole or in part, without the express permission of the faculty member.
Students are also forbidden to use their own means of recording any elements of an online class or lecture without express permission of the instructor. Any unauthorized sharing of course content may constitute a breach of the Academic Regulations.

Disabilities: Saint Mary's University is committed to providing reasonable accommodations for all persons with disabilities. Students with disabilities who need accommodations shall first contact the Fred Smithers Centre before requesting accommodations for this class.

Students who need accommodations in this course must contact the instructor in a timely manner (at least one week before examinations) to discuss needed accommodations.

Saint Mary's University

Department of Mathematics
& Computing Science

(Tentative and ambitious!) course calendar:

We will cover part of Chapter 2 and Chapters 3-4-5 of Braun's book, plus some additional notes.

Week	Topic	Important dates
1 (Jan 10th)	Welcome! 2.8 Series solutions (singular points, Euler's equation; regular singular points, method of Frobenius)	Jan 14th – course registration deadline
2 (Jan 17th)	2.8 Series solutions (Equal roots and roots differing by an integer) 3.1 Algebraic properties of solutions of linear systems. 3.2 Vector spaces. 3.3 Dimension of a vector space.	Jan 18th – course drop deadline
3 (Jan 24th)	3.4 Applications of linear algebra to differential equations. 3.5 The theory of determinants. 3.6 Solutions of simultaneous linear equations.	
4 (Jan 31th)	3.7 Linear transformations. 3.8 The eigenvalue-eigenvector method of finding solutions. 3.9 Complex roots. 3.10 Equal roots.	
5 (Feb 7th)	3.11 Fundamental matrix solutions; e^{At}. 3.12 The nonhomogeneous equation; variation of parameters.	
6 (Feb 14th)	4.1 Qualitative theory of differential equations. 4.2 Stability of linear systems. 4.3 Stability of equilibrium solutions.	
7 (Feb 21st)	**** Winter break ****	Feb 21st – *Heritage Day*
8 (Feb 28th)	(notes) Conservative Systems. 4.4 The phase-plane. 4.6 Qualitative properties of orbits. 4.7 Phase portraits of linear systems.	
9 (Mar 7th)	4.8 Long time behaviour of solutions; Poincaré-Bendixson Theorem. (notes) The Van der Pol oscillator.	
10 (Mar 14th)	4.9 Introduction to bifurcation theory. 4.10 Predator-prey problems. 4.11 The principle of competitive exclusion in population biology.	Mar 17th – course withdrawal deadline
11 (Mar 21st)	4.12 The Threshold Theorem of epidemiology (SIR model and COVID). (notes) A model on competitive markets. 5.1 Two point boundary–value problem.	

Week	Topic	Important dates
12 (Mar 28th)	5.2 Intro to PDEs. 5.3 The heat equation; separation of variables. 5.4 Fourier series. 5.5 Even and Odd functions.	
13 (Apr 4th)	5.6 Return to the heat equation. 5.7 The wave equation. 5.8 Laplace's equation.	

Disclaimer: the instructor reserves the right to make changes to the course outline and course content should this be necessary for academic or other reasons. Changes will also be posted on Brightspace and promptly communicated. Every effort will be made to minimize such changes.