1. Let $0 < \alpha < 1$. Prove that $\sqrt{\alpha} > \alpha$.

2. Prove that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.

3. Prove there are infinitely many primes.

   PART 1:
   Lemma. Any integer $n > 1$ has a prime factorization.
   (such a factorization is actually unique, but we won't prove it here)

   PART 2:
   Theorem. There exists infinitely many primes.

4. Show that if $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$, then $a + b \in \mathbb{Q}$.

5. If $n \in \mathbb{N}$ and $2^n - 1$ is prime, then n is prime.

6. If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd.

7. Prove that for $x,y \in \mathbb{R}$, $(x + y)^2 = x^2 + y^2$ if and only if $x = 0$ or $y = 0$.

8. A positive integer n is divisible by 3 if and only if the sum of the digits of n is divisible by 3.

9. What implications would we have to prove to prove the following theorem from linear algebra?

   Theorem. Suppose A is an $n \times n$ matrix. The following statements are equivalent:

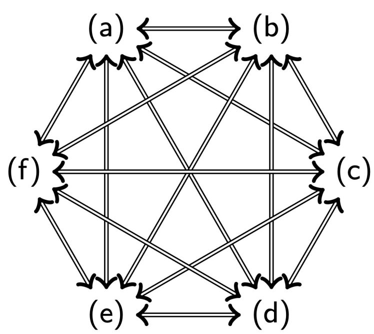
   (i) The matrix A is invertible.

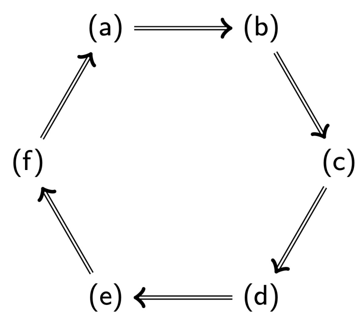
   (ii) The equation $A\vec{x} = \vec{b}$ has unique solution for every $\vec{b} \in \mathbb{R}^n$.

   (iii) The equation $A\vec{x} = \vec{0}$ has only the trivial solution.

   (iv) The reduced row echelon form of A is Id.

   (v) The matrix A does not have 0 as an eigenvalue.





10. Prove that for any $x,y \in \mathbb{R} \setminus \{0\}$, the following statements are equivalent:

    (i) $|x + y| = |x| + |y|$

    (ii) $xy = |xy|$

    (iii) $(x + y)^2 = x^2 + y^2 + 2|xy|$

    (iv) x and y have the same sign