1. Given $n \in \mathbb{N}$, define a recursive function f as follows:

$$f(0) = 1 \qquad f(1) = 3 \qquad f(n) = 2f(n-1) - f(n-2) \;\; \forall n \geq 2$$

   Prove that for all $n \geq 0$, $f(n) = 2n + 1$.

2. The number of ways to break a $2 \times n$ candy bar into 2×1 pieces is F_{n+1}.

Generalizing Theorems
Often the role of induction is to generalize theorems from cases with few elements (*say, two*) to an arbitrary but finite number of elements. Below are a number of results with which we've been familiar in the case of just two elements. Now we generalize:

Example Let $n \in \mathbb{Z}^+$.

- Suppose $a_1, a_2, \ldots, a_n$ are all even integers. Then $\sum_{i=1}^{n} a_i$ is even.

3. Suppose A and $B_1, B_2, \ldots, B_n$ are all sets. Then

$$A \cap \left(\bigcup_{i=1}^{n} B_i \right) = \bigcup_{i=1}^{n} (A \cap B_i)$$

4. Let $n \in \mathbb{Z}^+$ and let A be a finite set with n elements. Then $\mathcal{P}(A)$ has 2^n elements.

5. Prove that for all $n \in \mathbb{N}$, $1^2 + 2^2 + 3^3 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

6. The interior angle sum of a convex n-gon is $(n-2)\pi$.

7. Give a proof of De-Moivre's theorem

$$[\cos(x) + i\sin(x)]^n = \cos(nx) + i\sin(nx)\ .$$

8. Prove that $a^2 - 1$ is divisible by 8 for all odd integers a.