Sets and subsets

1. Prove that $14 \in \{4a + 7b \mid a,b, \in \mathbb{Z}\}$

2. Prove that $\{12a + 4b \mid a,b \in \mathbb{Z}\} = \{4c \mid c \in \mathbb{Z}\}$

3. Prove that $A \subseteq B$ if and only if $A - B = \varnothing$.

4. Let A,B,C be sets. Prove that $(A \cap B) - C = (A - C) \cap (B - C)$.

5. Prove that $A - (A - B) = A \cap B$.

6. Prove that for all $m,n \in \mathbb{Z}$, we have $\{x \in \mathbb{Z} : mn|x\} \subseteq \{x \in \mathbb{Z} : m|x\} \cap \{x \in \mathbb{Z} : n|x\}$.

7. Prove or disprove each of the following:

   (a) Let $A = \{6a + 4 \mid a \in \mathbb{Z}\}$ and $B = \{18b - 2 \mid b \in \mathbb{Z}\}$.

      i. $A \subseteq B$

ii. $B \subseteq A$

Cartesian product

8. (a) $\{1,2\} \times \{1,3,6\}$

   (b) $\{a,b\} \times \{c,d\}$

   (c) $[0,1] \times [0,1]$

   (d) $\mathbb{R} \times (2,4)$

   (e) $\{5\} \times \mathbb{R}$

9. If A and B are finite sets, then what can you say about $|A \times B|$?

10. Let A, B, C be sets. Prove that if $B \subseteq C$, then $A \times B \subseteq A \times C$.

11. Let A, B, C be sets. Prove or disprove that if $A \times B = A \times C$, then $B = C$.

12. Let A, B, C be sets. Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

13. Let A, B, C be sets. Prove that $A \times (B - C) = (A \times B) - (A \times C)$.

Power set

14. (a) Write all subsets of the set $A = \{0,1,2\}$.

    (b) Which of the following statements are true?

    i. $\{0\} \subseteq \mathcal{P}(A)$

    ii. $\{1,2\} \in \mathcal{P}(A)$

    iii. $\{\{0,1\}, \{1\}\} \subseteq \mathcal{P}(A)$

    iv. $\emptyset \in \mathcal{P}(A)$

    v. $\emptyset \subseteq \mathcal{P}(A)$

    vi. $\{\emptyset\} \in \mathcal{P}(A)$

    vii. $\{\emptyset\} \subseteq \mathcal{P}(A)$

    viii. $\{1, \{1\}\} \subseteq \mathcal{P}(A)$

    (c) Can you generalize to say how many subsets a set of n elements has?

15. Prove or disprove the following: if A and B are sets, then $\mathcal{P}(A) - \mathcal{P}(B) = \mathcal{P}(A - B)$.

16. Let A, B be sets. Prove that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

Indexed sets

17. Given the indexed sets, compute the unions and intersections:

    (a) $A_i = [0,i)$ for $i = 1, \ldots, n$

$$\bigcup_{i=1}^{n} A_i =$$

$$\bigcap_{i=1}^{n} A_i =$$

(b) $A_i = [i-1,i]$ for $i = 1,\ldots,n$

$$\bigcup_{i=1}^{n} A_i =$$

$$\bigcap_{i=1}^{n} A_i =$$

(c) $A_n = [0,\frac{1}{n}]$ for $i = 1,\ldots,n$

$$\bigcup_{n=1}^{\infty} A_n =$$

$$\bigcap_{n=1}^{\infty} A_n =$$

(d) $A_n = [0,\frac{n}{n+1}]$ for $i = 1,\ldots,n$

$$\bigcup_{n=1}^{\infty} A_n =$$

$$\bigcap_{n=1}^{\infty} A_n =$$